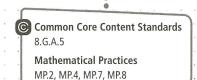


Lesson 6-9 Interior and Exterior Angles of Triangles



I can...

find the interior and exterior angle measures of a triangle.



Focus on math practices

Reasoning What assumption(s) did you need to make to find $m \angle 1$? Explain why your assumption(s) is reasonable. MP.2

Essential Question How are the interior and exterior angles of a triangle related?







EXAMPLE 1



Relate Interior Angle Measures in Triangles

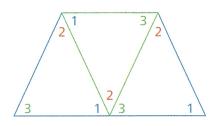




How can you describe the relationship between the three interior angles of each triangular tile in the backsplash?

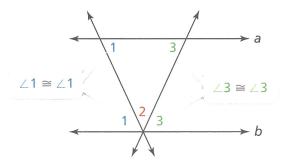


You can rotate and place the congruent tiles side-by-side to form the alternating pattern.



 $\angle 1$, $\angle 2$, and $\angle 3$ appear to fit together to form a line.

Use what you know about lines, transversals, and angle pair relationships to determine a relationship between the interior angles of a triangle.



Alternate interior angles are congruent, so $a \parallel b$. Since $\angle 1$, $\angle 2$, and $\angle 3$ form line b, a straight angle, $m\angle 1 + m\angle 2 + m\angle 3 = 180^{\circ}$.

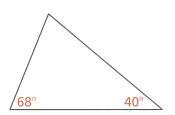
Generalize The sum of the measures of the interior angles of a triangle is 180°.

MR.8



Find the unknown angle measure in the triangle at the right.

Convince Me! Could a triangle have interior angle measures of 23°, 71°, and 96°? Explain.



EXAMPLE 2



Find Exterior Angle Measures







In the diagram, $m \angle 2$ is 68° and $m \angle 3$ is 42°. What is $m \angle 4$?

∠1 and ∠4 form a straight

 $\angle 1$, $\angle 2$, and $\angle 3$ are the angle and are supplementary. interior angles of a triangle.

$$m \angle 1 + m \angle 4 = 180^{\circ}$$

$$m \angle 1 + \underline{m} \angle 2 + \underline{m} \angle 3 = 180^{\circ}$$

 $m \angle 4 = m \angle 2 + m \angle 3$

$$m \angle 4 = m \angle 2 + m \angle 3$$

$$= 68^{\circ} + 42^{\circ}$$

$$m/4 = 110^{\circ}$$

For each exterior angle of a triangle, the two nonadjacent interior angles are its remote interior angles.

An exterior angle of a triangle is an angle formed by a side and an extension of an adjacent side.

Generalize The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles. © MP.8

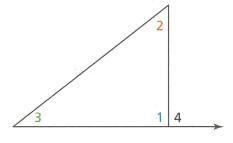


EXAMPLE 3 Use Algebra to Find Unknown Angle Measures

In the diagram, $m \angle 4$ is $(7x + 7)^\circ$, $m \angle 2$ is $(4x + 4)^\circ$, and $m \angle 3$ is $(4x - 9)^\circ$. What are $m \angle 4$ and $m \angle 1$?

Look for Relationships How

could you write an algebraic expression to represent $m \angle 1$? MP.7



STEP 1 Find the value of x.

$$m \angle 4 = m \angle 2 + m \angle 3$$

$$(7x + 7)^{\circ} = (4x + 4)^{\circ} + (4x - 9)^{\circ}$$

$$7x + 7 = 8x - 5$$

$$7x + 7 - 7x = 8x - 5 - 7x$$

$$7 = x - 5$$

$$12 = x$$

STEP 2 Find $m \angle 4$.

$$m \angle 4 = (7x + 7)^{\circ}$$

$$= 7(12) + 7$$

$$= 84 + 7$$

$$m \angle 4 = 91^{\circ}$$

STEP 3 Find $m \angle 1$.

$$m \angle 4 + m \angle 1 = 180^{\circ}$$

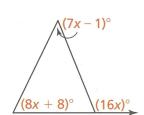
$$91^{\circ} + m \angle 1 = 180^{\circ}$$

$$91^{\circ} + m \angle 1 - 91^{\circ} = 180^{\circ} - 91^{\circ}$$

$$m \angle 1 = 89^{\circ}$$



What is the measure of the exterior angle shown?



KEY CONCEPT



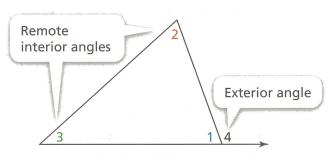


The sum of the measures of the interior angles of a triangle is 180° .

$$m \angle 1 + m \angle 2 + m \angle 3 = 180^{\circ}$$

The measure of an exterior angle of a triangle is equal to the sum of the measures of its remote interior angles.

$$m \angle 2 + m \angle 3 = m \angle 4$$



Do You Understand?

1. ? Essential Question How are the interior and exterior angles of a triangle related?

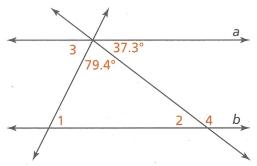
2. Reasoning Maggie draws a triangle with a right angle. The other two angles have equal measures. What are the possible values of the exterior angles for Maggie's triangle? Explain.

MP.2

3. Brian draws a triangle with interior angles of 32° and 87°, and one exterior angle of 93°. Draw the triangle. Label all of the interior angles and the exterior angle.

Do You Know How?

Use the diagram below for 4 and 5. Assume that $a \parallel b$.



4. What are the measures of $\angle 1$ and $\angle 2$? Explain.

5. What are the measures of $\angle 3$ and $\angle 4$? Explain.

6. In $\triangle ABC$, $m \angle A = x^{\circ}$, $m \angle B = (2x)^{\circ}$, and $m \angle C = (6x + 18)^{\circ}$. What is the measure of each angle?



TUTORIAL

Practice & Problem Solving



Scan for Multimedia

7. Leveled Practice For the figure shown, find $m \angle 1$.

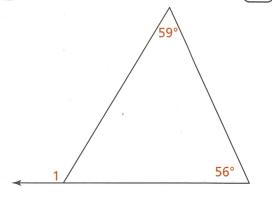
Angle 1 is an

angle of the triangle.

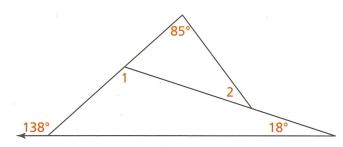
 $m \angle 1$ is equal to the sum of its

$$m \angle 1 = \bigcirc^{\circ} + \bigcirc^{\circ}$$

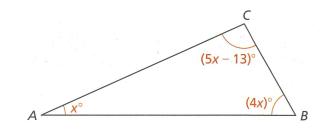
$$m \angle 1 = \bigcap^{\circ}$$



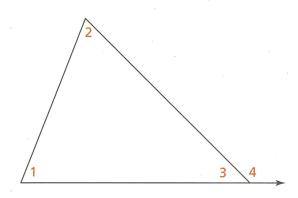
8. Find $m \angle 1$ and $m \angle 2$.



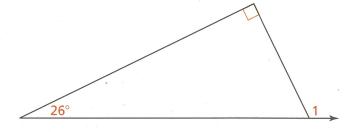
9. In $\triangle ABC$, what is $m \angle C$?



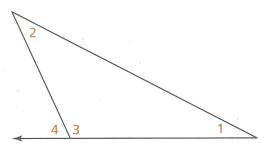
10. In the figure, $m \angle 1 = (8x + 7)^\circ$, $m \angle 2 = (4x + 14)^\circ$, and $m \angle 4 = (13x + 12)^\circ$. Your friend incorrectly says that $m \angle 4 = 51^\circ$. What is $m \angle 4$? What mistake might your friend have made?



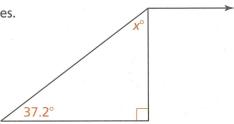
11. What is $m \angle 1$?



12. Higher Order Thinking Given that $m \angle 1 = (16x)^\circ$, $m \angle 2 = (8x + 21)^\circ$, and $m \angle 4 = (25x + 19)$, what is an expression for $m \angle 3$? What is $m \angle 3$?



13. A ramp attached to a building is being built to help with deliveries. The angle that the bottom of the ramp makes with the ground is 37.2°. Find the measure of the other acute angle.

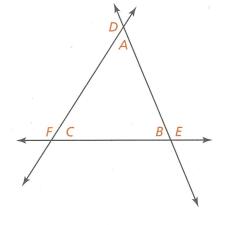


© Assessment Practice

14. What are the two remote interior angles for $\angle F$?

- \bigcirc $\angle C$ and $\angle B$
- B $\angle A$ and $\angle B$
- \bigcirc $\angle A$ and $\angle C$
- \bigcirc $\angle E$ and $\angle D$

358



15. In the figure, $m \angle 1 = (3x + 12)^\circ$, $m \angle 2 = (3x + 18)^\circ$, and $m \angle 3 = (7x + 10)^\circ$. What is $m \angle 3$? Explain your method.

